Online and Stochastic Learning with a Human Cognitive Bias

Hidekazu Oiwa, Hiroshi Nakagawa
The University of Tokyo
General Setting

- Supervised learning to learn **linear predictor** $w \in \mathcal{W}$
  - to predict **label** $y \in \{-1, 1\}$ from **input** $x \in \mathcal{X}$
  - e.g. Spam Mail Filtering

- **Approach:** Sequential Learning
  - Update predictor each time algorithm receives one labeled datum
  - Advantageous to large-scale learning: Do **not** store processed data
  - e.g. Online / Stochastic Learning

![Diagram showing data and predictor over time]
Human Cognitive Bias

- When applying sequential learning to practical applications, conventional framework causes problem
- Sometimes algorithms misclassify data that were correctly classified in the past
- User utility may be crucially deteriorated in this case
- Utility Maximization ≠ Prediction Error Minimization
- We focus on sequential learning with human cognitive bias
Outline

• Endowment Effect as a human cognitive bias

• Empirical analysis of this effect toward utility maximization

• New framework: Online and Stochastic Learning with a Human Cognitive Bias

• Proposed Algorithm: Endowment-induced OGD

• Theoretical Analysis

• Experimental Analysis
Endowment Effect [Thaler+ 80]

- One of the major human cognitive biases
  - Human tends to pay more money in event A
    - A. Prevent loss of already possessed object
    - B. Buy a new object
  - Human utility changes even if the outcome is the same
In the notion of Sequential Learning ...

- Event A tends to decrease human utility
  - A. Misclassify data that we correctly predicted in the past
  - B. Misclassify unseen data
- Does this bias truly exist?
Experiment on Endowment Effect

- Verify endowment effect for human utility
- Utilize crowdsourcing system to assign tasks
- Set up scene recognition systems
- Assign several tasks to each worker

Training

Receive pictures and predicted labels
Send correct label if misclassified

Test

Receive new pictures and labels
Evaluate learnability at a five scale

Predicted label
Restaurant

Predicted label
Restaurant
Experiment on Endowment Effect

- We assign two type tasks to each user
  - (type-I) Same image does not appear in both phases
  - (type-II) Images are redisplayed. They are correctly classified in training phase but misclassified in test phase
- Misclassification rates are fixed in both types
- **Result:** Type-II has lower evaluation than type-I with 1% level of significance
- Endowment effect badly affect worker’s evaluation

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>type-I</td>
<td>2</td>
<td>3</td>
<td>15</td>
<td>73</td>
<td>7</td>
<td>3.80</td>
</tr>
<tr>
<td>type-II (duplicate)</td>
<td>2</td>
<td>11</td>
<td>26</td>
<td>54</td>
<td>7</td>
<td>3.53</td>
</tr>
</tbody>
</table>
Sequential Learning with Endowment Effect

• Define this human cognitive bias as divestiture loss

\[
\text{[Divestiture loss]} = \ell(w; z)1_{\text{prev}}
\]

• \( z = (x, y) \): Datum

• \( \ell(w; z) \): Loss function

• \( 1_{\text{prev}} \): This function becomes 1 when sample \( z \) was correctly classified in the past; 0 otherwise.

• We add this term to objective functions
  
  • Online learning: [Regret] + \( \gamma \)[Divestiture loss Regret]
  
  • Stochastic Learning: [Expected loss] + \( \gamma \)[Divestiture loss]
• Endowment Effect as a human cognitive bias
  • Empirical analysis of this effect toward utility maximization
• New framework: Online and Stochastic Learning with a Human Cognitive Bias
• Proposed Algorithm: Endowment-induced OGD
  • Theoretical Analysis
  • Experimental Analysis
Endowment-induced OGD (E-OGD)

- We propose a new algorithm based on Online Gradient Descent (OGD):

\[
\mathbf{w}_{t+1} = \Pi_{\mathcal{W}} \left( \mathbf{w}_t - \eta_t \nabla \ell(\mathbf{w}_t; z_t) \right)
\]

where \( \eta_t = \begin{cases} 
\frac{c(1 + \gamma)}{\sqrt{t}} & \text{if } \hat{y}_t = y_t \\
\frac{c}{\sqrt{t}} & \text{if } \hat{y}_t \neq y_t 
\end{cases} \)

- \( \gamma \geq 0 \) : parameter to adjust importance of endowment effect
- When \( \gamma = 0 \), E-OGD becomes normal OGD
- According to prediction result, adjust step width
- When current datum is correctly classified, update parameters aggressively
- This adjustment tends to prevent misclassification
Theoretical Analysis for Online Learning

- Regret Analysis
- New objective: \([\text{Regret}] + \gamma [\text{Divestiture loss Regret}]\)
- We proved E-OGD achieves \(O(\sqrt{T})\) upper bound
- The same rate as OGD for normal setting [Zinkevich+ 03]

**Theorem 3.** Let \(w_1, \ldots, w_{T+1}\) be derived according to E-OGD’s update rule. Assume that for all \(w_t\), \(\|w_t\|_2 \leq R\) and \(\|\nabla \ell_t(w_t)\|_2 \leq G\) are satisfied. If loss functions are convex and we set a sequence of step widths \(\eta_1:T\) as denoted above, the upper bound of regret is obtained by setting \(c = \sqrt{2R/G(1 + \gamma)}\) as follows:

\[
\text{Regret}(T) \leq 2\sqrt{2RG(1 + \gamma)}\sqrt{T}.
\]
Theoretical Analysis for Stochastic Learning

- Expected Loss Analysis
  - New objective: [Expected loss] + \( \gamma \) [Divestiture loss]
  - E-OGD achieves \( O\left(\frac{1}{\sqrt{T}}\right) \) upper bound under some assumptions
  - The same rate as OGD for normal setting [Cesa-Bianchi+ 04]

**Theorem 4.** Assume that the conditions in Theorem 3 are satisfied and there is an integer \( t_p \) such that \( r_t(z) = r_{t_p}(z) \) for any \( t \geq t_p \). In this setting, the following formula is satisfied for any \( u \in \mathcal{W} \).

\[
E_{D_T} \left[ E_{z \sim \mathcal{D}_P} [\ell(\bar{w}; z)] \right] - E_{D_T} \left[ E_{z \sim \mathcal{D}_P} [\ell(u; z)] \right] 
\leq \frac{\sqrt{2RG}(1 + \gamma)}{(\sqrt{T} - (t_p + 1)/\sqrt{T})/(2 - \sqrt{t_p - 1}/\sqrt{T})},
\]

where \( \bar{w} = \sum_{t = t_p}^{T} w_t / (T - t_p + 1) \).
Experiment

- Five binary classification tasks by one-pass stochastic learning
- Loss function: Logistic loss
- Learning rate: $\eta_t = \eta / \sqrt{t}$
- Constant factor is set to minimize the objective
- Trade-off parameter: $\gamma = 1$
- Evaluation: $[\text{Expected loss}] + \gamma [\text{Divestiture loss}]$

Table 2: Dataset Specifications. $T$ is the number of training data, $S$ is test data size, $N$ is the number of features.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>$T$</th>
<th>$S$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>news20</td>
<td>15,000</td>
<td>4,996</td>
<td>1,335,191</td>
</tr>
<tr>
<td>rcv1</td>
<td>20,242</td>
<td>677,399</td>
<td>47,236</td>
</tr>
<tr>
<td>algebra</td>
<td>8,407,752</td>
<td>510,302</td>
<td>20,216,830</td>
</tr>
<tr>
<td>BtA</td>
<td>19,264,097</td>
<td>748,401</td>
<td>29,890,095</td>
</tr>
<tr>
<td>webspm-t</td>
<td>315,000</td>
<td>35,000</td>
<td>16,609,143</td>
</tr>
</tbody>
</table>
### Experimental Result

<table>
<thead>
<tr>
<th></th>
<th>Loss Type</th>
<th>E-OGD</th>
<th>OGD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>news20</strong></td>
<td>Expected</td>
<td>$3.11 \times 10^{-2}$</td>
<td>$3.85 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>Divestiture</td>
<td>$1.37 \times 10^{-2}$</td>
<td>$2.20 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>Cumulative</td>
<td>$4.48 \times 10^{-2}$</td>
<td>$6.05 \times 10^{-2}$</td>
</tr>
<tr>
<td><strong>rcv1</strong></td>
<td>Expected</td>
<td>$3.53 \times 10^{-2}$</td>
<td>$3.80 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>Divestiture</td>
<td>$1.25 \times 10^{-2}$</td>
<td>$1.78 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>Cumulative</td>
<td>$4.79 \times 10^{-2}$</td>
<td>$5.58 \times 10^{-2}$</td>
</tr>
<tr>
<td><strong>algebra</strong></td>
<td>Expected</td>
<td>$3.35 \times 10^{-1}$</td>
<td>$3.13 \times 10^{-1}$</td>
</tr>
<tr>
<td></td>
<td>Divestiture</td>
<td>$5.89 \times 10^{-2}$</td>
<td>$1.02 \times 10^{-1}$</td>
</tr>
<tr>
<td></td>
<td>Cumulative</td>
<td>$3.94 \times 10^{-1}$</td>
<td>$4.16 \times 10^{-1}$</td>
</tr>
<tr>
<td><strong>BtA</strong></td>
<td>Expected</td>
<td>$3.29 \times 10^{-1}$</td>
<td>$3.11 \times 10^{-1}$</td>
</tr>
<tr>
<td></td>
<td>Divestiture</td>
<td>$7.64 \times 10^{-2}$</td>
<td>$1.17 \times 10^{-1}$</td>
</tr>
<tr>
<td></td>
<td>Cumulative</td>
<td>$4.05 \times 10^{-1}$</td>
<td>$4.28 \times 10^{-1}$</td>
</tr>
<tr>
<td><strong>webspam-t</strong></td>
<td>Expected</td>
<td>$2.62 \times 10^{-2}$</td>
<td>$2.75 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>Divestiture</td>
<td>$8.29 \times 10^{-3}$</td>
<td>$1.16 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>Cumulative</td>
<td>$3.45 \times 10^{-2}$</td>
<td>$3.91 \times 10^{-2}$</td>
</tr>
</tbody>
</table>
Conclusion

- Online and stochastic learning with a human cognitive bias
- Verifying endowment effect through subjective experiment
- Mathematical modeling of endowment effect
- E-OGD heals negative effect of divestiture loss
- Theoretical analyses of E-OGD
- E-OGD obtains better empirical performance